Proceeding of ICGPSH 2025 Symposium: Exploring Spelling Evolution and Educational Approaches to Dyslexia
DOI: 10.54254/2753-7048/2025.HT26139

Theoretical Connotation and Cultivation Pathway of 'Human-AI Collaborative Mathematical Literacy' in the Context of Artificial Intelligence

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Abstract: Artificial Intelligence (AI) is profoundly reshaping mathematical practice, revealing limitations in traditional individual-centered mathematical literacy. This paper proposes a new paradigm of 'Human-AI Collaborative Mathematical Literacy' (HACML) based on distributed cognition theory. The necessity stems from four challenges: overcoming biological cognitive limitations, adapting to AI-driven workflow restructuring, mitigating algorithmic ethical risks, and bridging educational gaps. HACML encompasses four core competencies: critical interaction, meta-tool cognition, collaborative decision-making, and ethical judgment, shifting the evaluation unit from individuals to 'human-AI system' collaborative efficacy. Cultivation pathways include: innovating three teaching principles (AI transparency, contextual immersion, reflective practice); designing implementation strategies for four-dimensional capabilities; and transforming teachers into 'cognitive ecology designers'. This provides a systematic framework for mathematics education in the AI era.

Keywords: Human-AI Collaborative Mathematical Literacy (HACML), Mathematics Education, Distributed Cognition

1. Introduction

The rapid development of artificial intelligence technology is triggering profound deconstruction and reconstruction of mathematical practice paradigms. Algorithmic decision-making permeates optimization of complex mathematical systems, automated proof tools subvert traditional discovery pathways, and big data modeling increasingly positions mathematics at the forefront of solving high-dimensional, real-time practical problems. This technological wave not only reshapes mathematical practice but also exposes deep limitations in traditional mathematical literacy theories.

Conventional models persist in an isolated 'human-centered' cognitive view, narrowly defining mathematical ability as symbolic manipulation and logical reasoning within individual minds. As Dewey critiqued: 'Traditional education treats knowledge as static possessions rather than dynamic tools of inquiry' [1]. Such frameworks fail to explain the distributed cognitive ecology formed when AI deeply embeds in teaching processes, where mathematical problem-solving increasingly relies on

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DOI: 10.54254/2753-7048/2025.HT26139

dynamic collaboration between humans and intelligent systems across data flows, algorithmic logic, and intuitive insights.

Literature further reveals practical dilemmas of traditional literacy paradigms: (1) Failure of personalized teaching - standardized instruction struggles with diverse cognitive styles, while AI-enabled personalized learning paths significantly enhance efficacy [2]; (2) Disembedded abstract knowledge - highly abstract courses like advanced algebra lack digital scaffolding, causing methodological deficiencies [3]; (3) Lagging evaluation mechanisms - traditional manual grading is inefficient, while 'human-AI collaboration' reduces costs while ensuring fairness. These challenges validate Bruner's view: 'The task of education is not to transmit closed knowledge but to cultivate abilities for cultural practice participation' [4]. When AI becomes a structural element of cognitive practice, mathematical literacy must evolve from 'individual computational ability' to 'collaborative efficacy of human-AI cognitive systems'.

This paper therefore proposes the concept of Human-AI Collaborative Mathematical Literacy (HACML), necessitated by three transformative trends: (1) Cognitive revolution - distributed cognition theory reveals intelligence resides in 'human-tool-environment' systems, requiring humans to harness AI to transcend brain limitations; (2) Social demands - risks of algorithmic hegemony necessitate citizens' critical ability to identify biases in mathematical models [5]; (3) Educational evolution - Freire's 'pedagogy of the oppressed' warns that technology without critical consciousness reinforces knowledge hegemony [6], while HACML cultivates mathematical agency in technological societies.

Guided by distributed cognition theory, this study reconstructs mathematical literacy paradigms, providing cognitive foundations and practical pathways for AI-era mathematics education. While responding to technological challenges, it calls for education's essential return. As Whitehead stated: 'Education is the art of utilizing knowledge' [7] - in the human-AI collaboration era, this art's core becomes wisdom in cognitive management.

2. Theoretical perspective: philosophical and cognitive foundations of human-AI collaboration

HACML's theoretical foundation lies in the Distributed Cognition paradigm proposed by cognitive scientist Edwin Hutchins. This theory fundamentally challenges traditional cognitive science that views cognition as isolated brain activity, instead conceptualizing cognition as dynamic information flow across biological brains, artificial tools, environmental symbol systems, and social collaboration networks [8]. Its core proposition states: intelligent behavior emerges not solely from biological brains but from system-level capabilities arising through collaborative activities among humans, technological tools, and contexts.

This perspective resonates profoundly with Vygotsky's sociocultural theory: 'Higher psychological functions first appear as social relations between individuals before internalizing as individual functions' [9]. When AI becomes a key node in cognitive ecologies, it essentially transforms into socially embedded 'cognitive artifacts' that reshape mathematical thinking pathways and boundaries through externalized cognitive processes [10].

In mathematics, the distributed cognition lens reveals essential characteristics of human-AI collaboration. Mathematical problem-solving no longer relies purely on individual mathematicians' internal mental resources but forms an extended 'cognitive assemblage' [11] through functional coupling between humans and AI systems, comprising three layers: (1) Symbolic layer - humans input mathematical language, AI outputs structured data/proof steps; (2) Computational layer - humans construct problem frameworks through intuition, AI performs pattern search and

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verification; (3) Metacognitive layer - humans conduct critical monitoring, AI provides real-time feedback.

This process validates Simon's assertion: 'Humans as bounded rationality systems must rely on external tools to transcend cognitive boundaries' [12]. Human-AI collaboration transfers high-dimensional computation and exhaustive verification tasks to AI, freeing human brain resources for creative conjectures and value judgments. Thus under distributed cognition, HACML evaluation shifts from individual knowledge storage to abilities in coordinating information flows, optimizing cognitive labor division, and stimulating emergent system efficacy within 'human-AI' systems [13].

3. Necessity of cultivating HACML: addressing triple challenges in the intelligent era

The proposal of HACML responds to systemic challenges in mathematics education brought by AI development. This new literacy paradigm, rooted in cognitive science frontiers, mathematical practice transformations, and socio-ethical demands, demonstrates applicability and necessity in Chinese K-12 education contexts.

3.1. Cognitive dimension: transcending biological brain limitations

While human brains excel at pattern recognition and abstract reasoning, biological limitations become apparent when confronting modern mathematics' complexity. Distributed cognition identifies three 'individual cognitive boundaries': (1) High-dimensional 'cognitive blind spots' - traditional geometry teaching relies on 2D/3D spatial imagination, while machine learning involves higher-dimensional data beyond direct human comprehension; (2) 'Overload dilemma' of massive information and complex reasoning - human working memory capacity bottlenecks when handling massive data or combinatorial problems like complex network analysis; (3) 'Insight limitations' regarding hidden patterns - non-intuitive correlations in big data often escape direct human observation.

As Perkins emphasizes 'beyond individual intelligence' [14], such boundaries require external tools for transcendence. AI serves as a 'cognitive enhancer' [15] that augments rather than replaces core human mathematical thinking (intuition, creativity, value judgment). Through functional coupling, it forms 'cognitive assemblages' with human brains to overcome dimensional, speed, and insight constraints. A core HACML objective is cultivating students' 'meta-ability' to effectively harness these cognitive enhancers.

3.2. Practical and social dimensions: responding to paradigm restructuring and ethical challenges

Mathematical research is shifting from 'intuition-proof' to 'conjecture-verification-optimization' collaborative cycles. Industrial applications integrate mathematical modeling with intelligent decision-making, requiring practitioners' 'meta-tool cognition' to understand model logic. Without updated paradigms, professionals cannot engage in contemporary research and industrial work.

Algorithmic society's ethical crises demand attention. Freire's warning - 'Technology devoid of critical consciousness becomes new tools of oppressive power' [6] - grows increasingly pertinent. Key algorithmic risks include: (1) Institutionalized bias and fairness deficits - algorithms may amplify historical biases in training data; (2) Responsibility ambiguity and traceability dilemmas - AI error attribution involves complex mathematical responsibility chains; (3) Information cocoons and cognitive narrowing - recommendation algorithms may constrain cognitive diversity. HACML

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aims to cultivate Giroux's ideal of 'critical users rather than compliant consumers of technology' [16].

3.3. Educational dimension: bridging curriculum-system and technological reality gaps

Current mainstream mathematics education in Chinese K-12 schools shows significant disconnects with cognitive transformations, practice restructuring, and social needs, manifesting as: (1) Outdated objectives - overemphasizing isolated problem-solving skills while neglecting systemic collaboration; (2) Content deficiencies - algorithmic principles and ethical education gaps creating 'algorithmic illiteracy'; (3) Monolithic methods - lacking authentic collaborative practice, violating situated cognition theory; (4) Evaluation biases - ignoring assessment of high-order collaborative abilities like dynamic cognitive labor division and ethical decision-making [17].

This disconnect reflects Goodlad's 'enormous gap between operational curriculum and needed experiential curriculum' [18]. Without timely intervention, mathematics education risks failing its cultivation objectives, losing social recognition, and forfeiting legitimacy in preparing future citizens.

4. Theoretical construction of HACML: from individual mind to cognitive ecology

Based on distributed cognition, HACML denotes individuals' abilities to coordinate resources, optimize labor division, and assume ethical responsibilities within 'human-AI' cognitive assemblages, marking a paradigm shift from 'individual computational ability' to 'system collaborative efficacy'. HACML comprises four interconnected core dimensions:

4.1. Critical interaction competence: foundation of cognitive autonomy

Critical interaction requires learners to actively question, examine, and deconstruct AI outputs and decision processes rather than passively accepting them as 'authoritative answers'. Its theoretical foundation lies in Freire's critical pedagogy: 'True education awakens critical consciousness, transforming subjects into reflective world-transformers rather than passive adapters' [6]. In Chinese classrooms, this manifests as: identifying data biases/limitations; scrutinizing algorithmic logic and implicit assumptions; questioning result reliability and applicability.

4.2. Meta-tool cognition: cognitive schemata for harnessing technology

Meta-tool cognition transcends basic AI operation skills, requiring deep understanding of underlying mathematical principles, capability boundaries, and system vulnerabilities. This extends Bruner's 'tool mediation theory': technological tools fundamentally reshape cognitive processes themselves [4]. AI's internal mechanisms directly influence how we think about, express, and solve mathematical problems.

4.3. Collaborative decision-making: co-evolution of cognitive ecology

This dimension focuses on dynamic allocation and optimization of cognitive loads in human-AI collaboration. Learners must intelligently delegate tasks based on respective human-AI strengths while maintaining effective information exchange and feedback loops, becoming 'cognitive ecology managers'. This extends Vygotsky's Zone of Proximal Development (ZPD) theory: AI serves as the

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'more capable interactant' [9], with humans leading problem framing and ethical weighing while AI handles data processing and complex computations.

4.4. Ethical judgment: responsibility anchor in technological society

Ethical judgment requires learners to anticipate and evaluate social impacts when applying mathematics and AI, grounded in Dewey's democratic education view: education must cultivate citizens' sensitivity to consequences of social actions [1]. This includes: anticipating potential risks/biases; assessing fairness/inclusion; defining responsibility; promoting corrections. Developing this capability liberates mathematics from 'instrumental rationality' constraints while cultivating responsible digital citizens.

4.5. Fundamental distinctions from traditional mathematical literacy

HACML's four-dimensional model shifts evaluation units from individuals to human-technology systems; focus from computational speed to systemic collaboration; extension to technical understanding and ethical responsibility; and value orientation toward social welfare. This model not only addresses technological challenges but promotes mathematics education's essential return. Its ultimate objective is cultivating modern citizens capable of effective, just, and responsible thinking and action using mathematical reasoning in complex, intelligent worlds.

5. Cultivation pathways for HACML: grounded in theory, rooted in local practice

Cultivating HACML requires systematic transformation of teaching practices - a systems engineering project integrating pedagogy, cognitive science, and technology philosophy. Drawing on Shulman's Pedagogical Content Knowledge (PCK) theory [19], we construct a three-dimensional cultivation framework.

5.1. Teaching principle innovation: three pillars for human-AI collaborative cognitive ecology

HACML cultivation necessitates fundamental shifts in teaching philosophies:

AI Transparency Principle: Demystifying black boxes to build cognitive trust. Following Mayer's multimedia learning principles [20], teaching should reveal AI's core logic and operational mechanisms. Chinese classroom practices may implement Perkins' 'understandable technology' concept [14] through visualizations of AI processes, explanatory domestic AI tools, and simplified analogical models.

Contextual Immersion Principle: Activating human-AI collaboration through authentic, complex problems. Knowledge develops through situated practice [21]. HACML's core value lies in solving real-world problems, requiring mathematics learning in authentic, ill-structured, interdisciplinary contexts (e.g., life-based PBL projects, real data analysis, simulated industrial/research workflows). VR-enhanced immersive mathematical inquiry environments effectively develop spatial thinking [22], while AI-driven vocational training models validate this principle's efficacy [23].

Reflective Practice Principle: Catalyzing metacognitive evolution. HACML formation requires iterative reflection. Structured reflection components should be embedded in human-AI learning activities, guiding students beyond 'what' and 'how' to 'why', 'effectiveness', 'limitations', and 'improvement'. Strategies include reflective journals, discussion protocols, and teacher modeling of reflective practice.

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5.2. Strategy system design: translating four-dimensional competencies into classroom practice

Concrete teaching strategies should implement HACML's four core dimensions:

Critical Interaction: Conduct algorithm auditing workshops using familiar algorithmic applications, guiding deep analysis of principles, data sources, and potential biases. Case selection should align with national conditions and student cognition levels, prioritizing educational and daily-life scenarios.

Meta-Tool Cognition: Implement 'mathematics-code/operation' dual-track teaching. Use platforms like Scratch/Mind+ to demonstrate algorithms as stepwise instructions. Design boundary-exploration tasks using encapsulated AI tools (e.g., inputting incomplete/extreme data) to reveal limitations. Connect mathematical principles, technical implementation, and actual outcomes.

Collaborative Decision-Making: Conduct structured task analysis and role-playing based on Sweller's cognitive load theory [24]. Design clear 'collaboration protocols' for task handover, information transfer, and feedback reception. Encourage dynamic adjustment of labor division during implementation, followed by reflective evaluation.

Ethical Judgment: Introduce concrete ethical dilemmas using student-relevant algorithm scenarios (e.g., AI classroom management). Organize multi-role simulations representing stakeholders, analyzing benefits/risks, assessing transparency, and discussing responsibility. Facilitate debates or hearings to integrate perspectives toward fairer solutions, as demonstrated in language education using ChatGPT [25].

5.3. Teacher role transformation: from knowledge transmitters to cognitive ecology designers

Teachers are pivotal for HACML implementation, requiring transformation from 'knowledge authorities' to 'designers', 'collaborators', and 'facilitators' of human-AI cognitive ecologies. This demands new professional competencies:

HACK Knowledge Framework: Dynamically integrating three core knowledges: (H) Human: deep mathematical content knowledge; (A) AI: understanding AI fundamentals, boundaries, applications, and ethics; (C) Collaborative: pedagogical strategies for designing human-AI learning activities. This extends traditional TPACK [26], emphasizing holistic design of 'human-AI cognitive collaboration systems'. Recent research confirms the necessity of dynamically expanding teacher knowledge frameworks [27].

Dynamic Cognitive Scaffolding: Teachers should provide 'just-in-time retreating support' [28] across phases: (Initial) Structured supports (curated AI tools, predefined frameworks); (Intermediate) Increased openness (student-selected tools, self-negotiated protocols); (Advanced) Student autonomy (self-defined problems, independent collaboration designs). Teachers should leverage group work, tiered tasks, and technology platforms for differentiated support.

Ethical Agency: Teachers must embody strong technological ethics. When using AI tools, they should demonstrate critical thinking and ethical considerations. Strictly evaluate AI products for data privacy, algorithmic transparency, potential biases, and educational ethics. Resist using AI for excessive student monitoring or test-prep efficiency at the expense of thinking cultivation. Advocate for transparent, fair, and responsible educational AI within professional capacity.

In China's educational digital transformation, teachers' ethical agency is crucial for ensuring healthy technology adoption, protecting minors, and cultivating socially responsible digital citizens.

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6. Conclusion

The proposed HACML concept responds to AI's profound reshaping of mathematical practice and cognitive ecology. Grounded in distributed cognition theory, it signifies a pivotal innovation in mathematical literacy paradigms, emphasizing system efficacy within 'human-AI' cognitive assemblages through integrated critical interaction, meta-tool cognition, collaborative decision-making, and ethical judgment.

This new paradigm drives three fundamental transformations in mathematics education: (1) Objective shift-from individual mastery of static knowledge to managing, coordinating, and innovating in dynamic human-AI cognitive ecologies; (2) Content expansion - from pure mathematical knowledge/skills to algorithmic philosophy, human-AI interaction strategies, and data ethics; (3) Power redistribution - from teacher-centered authority toward democratic negotiation and co-exploration within clear AI boundaries and ethical responsibilities.

Limitations include AI technology's rapid evolution continuously challenging and reshaping human-AI collaboration's connotations and boundaries, requiring theoretical frameworks and practical pathways to maintain openness and adaptability.

Future research must closely track technological frontiers, deepen understanding of human-AI cognitive collaboration mechanisms, and validate cultivation models across broader educational contexts to empower learners' mathematical existence and development in an intelligent era.

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