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# *Strategies for Cultivating High School Students' Mathematical Abstraction Ability under the New Curriculum Standards: A Case Study of Inequalities*

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***Abstract:*** Under the guidance of the new curriculum standards, mathematical abstraction ability is a critical aspect of developing students' core mathematical literacy. Taking inequalities as an example, this paper explores effective strategies for cultivating high school students' mathematical abstraction ability. The strategies are based on practical teaching examples of inequalities and include guiding students to understand abstract concepts, strengthening the expression of mathematical language, expanding the breadth and depth of students' mathematical thinking, and fostering students' ability for self-reflection. These strategies aim to help students understand and master the abstract concepts of inequalities, further understand abstract concepts in high school mathematics, improve students' mathematical literacy, and enhance their problem-solving abilities.

***Keywords:*** Mathematical abstraction, inequalities, teaching strategies, high school mathematics

## 1. Introduction

With the new curriculum standards focusing on developing students' core mathematical competencies—such as mathematical abstraction, logical reasoning, mathematical modeling, intuitive imagination, mathematical computation, and data analysis [1]—cultivating students' mathematical abstraction ability has become one of the key objectives of high school mathematics education. Mathematical abstraction ability refers to the process of abstracting quantitative relationships and spatial forms to create mathematical objects of study. As such, mathematics is not merely an abstract representation in the mind; more importantly, it is about how to transform abstract thinking into verbal or written expression. This ability is not only the foundation of mathematics as a discipline but also a reflection of students' overall cognitive development.

In this context, high school students' mathematical abstraction ability can help them gradually extract general rules and abstract concepts from the process of solving specific problems. Inequalities, as one of the common and important learning content in high school mathematics, provide a good opportunity for improving students' mathematical abstraction abilities. Through learning inequalities, students can train their abstract thinking abilities while understanding and mastering the basic properties and applications of inequalities, thereby enhancing their mathematical abstraction literacy and developing their understanding of the nature of mathematical objects.

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Inequalities are involved in various aspects of high school mathematics, including algebra, inequality transformations, optimization problems, monotonicity, and more, making it a highly integrated topic. This paper aims to explore how, under the guidance of the new curriculum standards, inequalities as a specific teaching content can be used, through designed teaching strategies, to effectively improve high school students' mathematical abstraction ability and prepare them for their future academic development.

## 2. The Importance of Mathematical Abstraction Ability in High School Mathematics Education

Mathematical abstraction thinking is a crucial foundation for students to develop rational thinking throughout their high school education. The quality of mathematical thinking refers to the characteristics and features that individuals exhibit during the process of mathematical thinking, including abstraction, rigor, systematics, coherence, flexibility, profundity, criticality, and creativity [2]. For example, when students begin their high school studies, they encounter functions, particularly abstract functions, which require them to understand and master the abstract concepts within functions. This enables them to abstract mathematical models from real-world problems, simplify them, and solve mathematical issues effectively. The process of cultivating students' mathematical abstraction ability also helps to develop their logical reasoning skills and mathematical thinking in everyday life, thereby promoting their overall development.

In recent years, the Chinese National College Entrance Examination (Gaokao) has increasingly emphasized the comprehensiveness of mathematics questions, focusing more on assessing students' mathematical thinking. The focus has gradually shifted from "solving problems" to "how to solve problems." Compared to high school mathematics, junior high school mathematics is more concrete and intuitive. As a result, students often experience difficulty when transitioning into high school mathematics.

For instance, in junior high school, students learn functions such as linear functions, inverse proportional functions, and quadratic functions, which are used to intuitively describe the relationship between two variables. The learning primarily focuses on the relationship where the dependent variable $y$ changes in response to the independent variable $x$ in a simple one-to-one correspondence. In contrast, high school mathematics introduces a new variable, the "parameter," in the second chapter on "Inequalities." This presents a challenge for students who, having only learned to solve problems in junior high without deeply understanding the connections between knowledge points, are often unsure of how to approach the parameter and may mistakenly confuse it with the independent variable.

Mathematical abstraction literacy generally develops in two stages: realistic abstraction and logical abstraction. The process of forming realistic abstraction literacy refers to students' learning of theoretical knowledge—mastering basic symbols and concepts while preparing for the conversion of mathematical knowledge [3]. Mathematical abstraction thinking helps students move from simply memorizing mathematical formulas and problem-solving methods to a deeper understanding of the essence of mathematical concepts. It enables them to generalize encountered mathematical problems, apply learned knowledge more flexibly, and simplify complex issues. In turn, this cultivates students' logical rigor and critical thinking, allowing them to view mathematical problems from a higher perspective, thus enhancing their interest in learning mathematics and their ability for independent learning. These abilities are essential qualities and key competencies needed in today's society.

Therefore, it is crucial for teachers to foster the mathematical abstraction abilities of students when they first enter high school. Such abilities not only help students understand and apply mathematics but also lay a solid foundation for their logical thinking and problem-solving skills during their high school years.

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## 3. Strategies for Cultivating Abstraction Ability

### 3.1. Teaching Strategies for Inequalities in Practical Instruction

The study of inequalities is introduced in the junior high school curriculum, and teachers can strengthen the connection between junior and senior high school knowledge. By integrating the inequalities learned in junior high school with the teaching content in high school, teachers can further enhance students' understanding of the mathematical knowledge system. In junior high school, students are exposed to inequalities such as linear inequalities, systems of inequalities, comparing functions through graphical representation, and solving optimization problems.

**Example 1:** As shown in Figure 1, the graph of the linear function $y_1 = x + 1$ intersects with the graph of the inverse proportional function $y_2 = \frac{2}{x}$ at points $A(1,2)$ and $B(-2,-1)$. Based on the graph, directly write the range of $x$ that satisfies $x + 1 < \frac{2}{x}$.

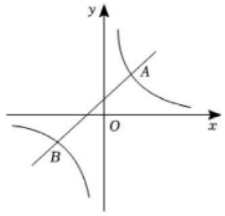



Figure 1: Graphs of $y_1 = x + 1$ and $y_2 = \frac{2}{x}$ and their intersection points

**Solution:** As shown in Figure 1, $0 < x < 1$ or $x < -2$.

In this question, the teacher can guide students to analyze the problem by referring to the graph. The graphical representation helps to visually demonstrate the concept of inequalities. The teacher can first review the relationship between the coordinate plane and functions, leading to the understanding that, for the same $x$-coordinate, the larger $y$-value corresponds to a larger function value. This means that the graph of the larger function value lies above the graph of the smaller function value, and the teacher can then proceed to determine the range of $x$.

In high school, the first type of inequality students encounter is the basic inequality. The textbook provides four different methods for solving it. The first method uses a geometric visualization in a circle to show the relationship between $\sqrt{ab}$ and $\frac{a+b}{2}$. By applying the familiar principle of "the hypotenuse is the longest side in a right triangle," the teacher can intuitively demonstrate the relationship between $\sqrt{ab}$ and $\frac{a+b}{2}$. The second and third methods approach the inequality algebraically, using the method of completing the square. The second method transforms the inequality into a perfect square form, while the third method uses the identity $(x + y)^2 = x^2 + y^2 + 2xy \geq 0$, setting $x = \sqrt{a}$ and $y = \sqrt{b}$, from which the basic inequality can be derived. The fourth method employs reverse reasoning, starting from the conclusion and working backward step-by-step to deduce that $\left(\sqrt{a} - \sqrt{b}\right)^2 \geq 0$.

In practical teaching, teachers can provide appropriate prompts and guide students to abstract mathematical models from problems, helping them understand the concept of mathematical abstraction and thus improving their mathematical abstraction ability.

After studying the basic inequalities, students then proceed to learn quadratic equations and inequalities from the perspective of functions.

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At the beginning of the lesson, the teacher should lead students in reviewing the quadratic functions learned in junior high school. Through specific examples, students can be guided to draw the graphs of quadratic functions, observe their characteristics such as the direction of the opening, vertex, axis of symmetry, and points of intersection with the $x$-axis. The teacher can then connect the functions to equations: if the quadratic function $y = ax^2 + bx + c$ intersects the $x$-axis at two points, one point, or not at all, these correspond to the three cases of solutions for the quadratic equation $ax^2 + bx + c = 0$, helping students further understand the geometric meaning of equations. Additionally, teachers can use dynamic demonstrations with software like Geometer's Sketchpad to show the effects of adjusting different parameters on the graph, helping students better understand the relationship between functions and equations.

Once students grasp the relationship between functions and equations, the teacher can introduce the geometric meaning of inequalities. By using the coordinate plane, the graph of the quadratic function can be divided into the upper and lower halves by the $x$-axis. The upper part corresponds to $y > 0$, and the lower part corresponds to $y < 0$. By identifying the ranges of $x$ for both parts of the graph, students can find the solution set of the equation.

**Example 2:** Solve the inequality $-x^2 + 11 < -2x - 4$.

**Method 1**

Move terms to one side:

$$x^2 - 2x - 15 > 0,$$

Factorize:

$$(x + 3)(x - 5) > 0,$$

This implies that $x + 3$ and $x - 5$ must have the same sign, so we have two cases:

1.

$$x + 3 > 0 \text{ and } x - 5 > 0,$$

2.

$$x + 3 < 0 \text{ and } x - 5 < 0,$$

Solving these gives:

$$x < -3 \text{ or } x > 5.$$

**Method** 2

Let

$$y = x^2 - 2x - 15,$$

then the graph of the function is shown in Figure 2.

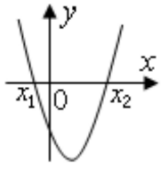



Figure 2: Graph of $y = x^2 - 2x - 15$

Here, the roots are $x_1 = -3, x_2 = 5$.

To solve $x^2 - 2x - 15 > 0$, we need to find where $y > 0$, which corresponds to the portion of the graph above the $x$-axis. This gives:

$$x < -3 \text{ or } x > 5.$$

Mathematical abstraction requires students to deduce general rules from specific mathematical problems and express and reason with mathematical symbols and language independently. Inequalities, in essence, are abstract mathematical concepts that connect two expressions with inequality signs (e.g., >, <, ⩾, ⩽). High school students must develop the ability to understand the

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properties and forms of inequalities, progressing from comparing the magnitudes of two numbers to solving linear inequalities, quadratic inequalities, parameterized inequalities, and eventually proving inequalities in derivative problems. Through this process, students gradually master the abstract meaning of inequalities and other concepts. As discussed above, when learning to solve quadratic inequalities, students first need to review the characteristics of quadratic function graphs learned in junior high school. Using the graph to find the solution set of the inequality will help convey the method and idea to students, fostering their ability to abstract concepts and further develop their mathematical thinking.

Once students are proficient in solving standard quadratic inequalities, learning to solve parameterized quadratic inequalities will deepen their understanding of the properties and applications of inequalities, laying a solid foundation for future mathematical learning. A parameterized quadratic inequality involves an inequality in which the quadratic expression includes parameters. The basic method for solving these inequalities is to separate the parameters and solve for the solution set under different parameter values.

When learning parameterized inequalities, students must first understand the abstract concept of "parameters" and how varying parameters affect the solution set of the inequality. Teachers can guide students to abstract the mathematical model in parameterized problems, thereby improving their ability to solve abstract problems through logical reasoning. This process helps students enhance their understanding of the range of parameters and how to handle parameters, ultimately internalizing their mathematical abstraction abilities.

Parameterized quadratic inequalities introduce high school students to the concept of inequalities. Through effective teaching, students not only learn to solve inequalities but also build a solid foundation for studying derivatives. More importantly, this fosters the development of students' mathematical abstraction and problem-solving abilities. Therefore, in the teaching process, teachers should purposefully nurture students' abstract mathematical thinking and employ various teaching methods to stimulate students' interest in learning mathematics and promote their overall development.

In high school mathematics, there is a close relationship between functions, derivatives, and inequalities. For example, inequalities are used to find the extrema and critical values of functions, assess the monotonicity of functions, and prove functional inequalities. By accurately grasping the essence of the problem and key information from the given conditions, students can abstractly construct the inequality relationships, apply known conditions and the properties of inequalities for reasoning and proof, and logically navigate the steps to solve mathematical problems. This requires students to possess strong mathematical abstraction and logical reasoning skills, and to flexibly apply the mathematical knowledge and techniques they have learned to solve problems. By solving mathematical problems with inequalities, students can significantly enhance their abstract thinking abilities and mathematical reasoning, improving their core mathematical literacy.

### 3.2. Strategies for Integrating Mathematical Abstraction into Classroom Teaching

1. Guide Students to Understand Abstract Concepts When Introducing Them: The process of discovering and understanding truth often requires learners to transition from vivid, intuitive images to abstract thinking, and then from abstract thinking to practical application [3]. The teaching goal should be student-centered, focusing on what students are able to do rather than what the teacher aims to teach them. For example, instead of saying “to cultivate students' ability to do something,” it should be framed as “students will be able to do something” [4]. The formation of mathematical abstract thinking is a long-term process. In actual teaching, teachers can use common real-world phenomena to help students quickly grasp abstract concepts. For instance, when teaching linear functions for the first time, teachers can relate them to the relationship between the side length and perimeter of a square, or the relationship between speed and time, helping students understand the definition and

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properties of abstract functions through concrete examples. During the process of guiding students from the concrete to the abstract, teachers should gradually increase the level of abstraction to avoid overwhelming students with concepts that are too advanced.

2. Strengthen the Expression of Mathematical Language: The new curriculum standards explicitly state that students of the new era should "be able to express the world using the language of mathematics." Teachers should integrate mathematical terminology and expressions into classroom teaching to improve students' ability to express and understand mathematics. For example, when teaching the relationships between two lines in middle school, teachers should emphasize definitions like point, line, parallel, and perpendicular, ensuring that students master and can accurately express these concepts using mathematical language. This will help students understand the relationships between lines and planes, or between planes themselves, more quickly when they encounter them in high school. By effectively linking new knowledge with prior knowledge, students can form a clear mental framework for understanding abstract concepts and become proficient in applying them [5]. Additionally, teachers can innovate in their teaching methods. For instance, in their spare time, students can be asked to abstract mathematical relationships from practical application problems from different angles and express the same relationships using different mathematical forms. Teachers may also guide students to convert one symbolic form into another, such as linking functions and equations. Strengthening the expression of mathematical language will help students better understand the logical relationships in mathematics, enhancing their mathematical abstraction skills and core mathematical literacy.

3. Expand the Breadth and Depth of Students' Mathematical Thinking: Mathematics teachers can raise open-ended questions in their teaching to spark students' curiosity and desire for knowledge. The exploration of content should cater to the needs of students at different levels, so teachers should consider student differences and conduct differentiated mathematical exploration activities [6]. This encourages students to think deeply, explore mathematical problems from different perspectives and levels, and fosters critical thinking and creativity. Open-ended questions require students to abstract general mathematical principles and concepts from specific problems. Through this process, students can identify general mathematical patterns, deepening their understanding of mathematical concepts and principles, and forming a more solid mathematical knowledge system. Moving from specific problems to general rules, and from general rules to practical application, this process can effectively cultivate students' ability to apply and innovate in mathematics, fulfilling the new curriculum's goal of "viewing and solving problems from a mathematical perspective." For example, in the classroom teaching of functions, teachers can first ask students to independently plot a function graph using the "five-point method" and explore the commonalities and differences between the graphs of different functions, deepening their understanding of the properties of function graphs.

4. Cultivate Students' Ability to Reflect Independently: Reflection and summarization help students deepen their understanding of previously learned concepts and lay a solid foundation for future learning. Reflection should first involve reviewing the abstract concepts learned in class and the examples provided by the teacher on how to understand those abstract concepts. Based on this reflection, students can be guided to understand the specific mathematical principles underlying these abstract concepts. For example, when studying linear functions, students should recognize that the change in the dependent variable $y$ as the independent variable $x$ increases or decreases is fundamentally related to the principle of monotonicity behind the function. Additionally, students should reflect on their learning methods, attitudes, and their level of mastery of various concepts. Through communication with teachers, students can adjust their learning strategies in a timely manner, enhancing their understanding of weaker areas. This helps students abstract knowledge from textbooks and transfer it to real-life applications, fostering their ability to learn independently and creatively.

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## 4. Conclusion

The development of mathematical abstraction skills is a long-term process. In actual teaching, teachers should select appropriate teaching methods based on the specific needs of students. Teachers must grasp the essence and connotations of mathematics, highlight the intrinsic connections between mathematical concepts, encourage students to discover the beauty of mathematics, and stimulate their interest in learning the subject. This approach ensures the comprehensive development of students.

## References

[1] *Ministry of Education of the People's Republic of China. (2018). General high school mathematics curriculum standards (2017 edition). Beijing: People's Education Press.*

[2] *Li, F., & Zhang, D. (2022). Approaches to improving high school students' mathematical abstraction literacy. Education Science Forum, (10), 52-55.*

[3] *Wang, K. (2023). Strategies for cultivating high school students' mathematical abstraction literacy under the perspective of deep learning. Asia-Pacific Education, (16), 61-63.*

[4] *Cheng, H., & Hao, Y. Y. (2022). Key points to focus on in instructional design guided by mathematical core literacy. Mathematics Bulletin, 61(04), 15-18, 21.*

[5] *Jiang, H. (2022). Strategies for cultivating students' mathematical abstraction literacy in concept teaching. National Common Language and Literature Teaching and Research, (06), 47-49.*

[6] *Yang, X., & Qian, S. Q. (2023). Research on the reflection of mathematical core literacy in the content of high school textbooks. Journal of Liaoning Normal University (Natural Science Edition), 25(03), 20-25, 41.*